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Comparing (1) and (4) and equating coefficients of same powers of δx we get

$$\frac{1}{\underline{|n}}\frac{d^ny}{dx^n}=\sum_{k=1}^{h=n}C_k\frac{1}{\underline{|k}}\frac{d^ky}{dt^k},$$

where C_k is the coefficient of $\overline{\delta x}^n$ in the expansion of

$$\left[\delta x\frac{dt}{dx}+\frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2t}{dx^2}+\cdots\right]^k.$$

This formula gives d^ny/dx^n in terms of d^ky/dt^k and d^kt/dx^k, which is not quite the formula asked for. Compare Greenhill's "Differential and Integral Calculus," p. 180.

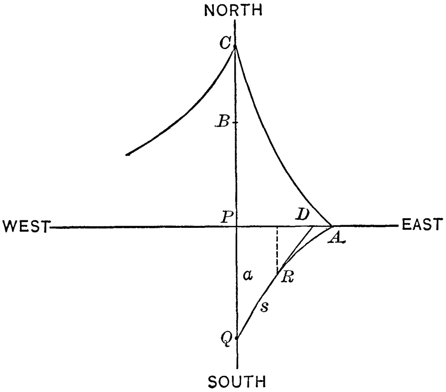
352. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

At point P there are n foxes. At Q, a rods south of P, there is a dog. The dog and the foxes are freed at the same instant and run at uniform speeds. Some of the foxes run east, some north, some west and some south. The dog runs first towards the foxes that ran east and always points toward them. He captures one of them and then instantly pursues the pack that ran north. In like manner, when he has captured one of them, he pursues those that ran west, then those that ran south, and then begins over again by pursuing the ones running east. If r is the ratio of the dog's speed to that of a fox, what is the total length of the n curves of pursuit.

(Generalization of a problem published in 1859 in the *Mathematical Monthly.*)

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Let S be the position of a fox at any time after the chase began and R the corresponding position of the dog. Taking PA for the x-axis and PQ for the y-axis we have



$$QR=r\cdot PD,\quad \text{i. e.,}\quad s=r\left(x-\frac{y}{\dfrac{dy}{dx}}\right)\quad\text{or}\quad \frac{ds}{dx}=r\left[1-\frac{\left(\dfrac{dy}{dx}\right)^2-y\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^2}\right].$$

Hence,

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=ry\frac{d^2y}{dx^2}\Big/\left(\frac{dy}{dx}\right)^2.$$

Let $\dfrac{dy}{dx} = p$; then $\dfrac{d^2y}{dx^2} = p\dfrac{dp}{dy}$.

Hence, $p\sqrt{1+p^2} = ry\dfrac{dp}{dy}$ and therefore $\dfrac{dy}{y} = \dfrac{rdp}{p\sqrt{1+p^2}}$.

Integrating,

$$\log cy = -r\log\frac{[1+\sqrt{1+p^2}]}{p}.$$

Hence,

$$cy^{1/r} = \frac{p}{1+\sqrt{1+p^2}} \qquad \text{or} \qquad c^2y^{2/r}(1+p^2) = p^2 - 2cpy^{1/r} + c^2y^{2/r}.$$

Hence,

$$\frac{dy}{dx} = \frac{2cy^{1/r}}{1-c^2y^{2/r}}. \tag{1}$$

Whence,

$$\frac{1-c^2y^{2/r}}{y^{1/r}}dy = 2cdx.$$

Integrating,

$$\frac{y^{-1/r+1}}{1-\frac{1}{r}} - c^2\frac{y^{1/r+1}}{1+\frac{1}{r}} = 2cx + c'. \tag{2}$$

When

$$x = 0, \qquad y = a, \qquad \frac{dy}{dx} = \infty.$$

Hence, from (1),

$$c = \pm\frac{1}{a^{1/r}},$$

and, from (2),

$$c' = \frac{a^{1-1/r}}{1-\frac{1}{r}} - \frac{1}{a^{2/r}}\frac{a^{1+1/r}}{1+\frac{1}{r}}.$$

Hence, from (2),

$$\pm\frac{2x}{a^{1/r}} = \frac{y^{1-1/r}}{1-\frac{1}{r}} - \frac{1}{a^{2/r}}\frac{y^{1+1/r}}{1+\frac{1}{r}} - \frac{a^{1-1/r}}{1-\frac{1}{r}} + \frac{1}{a^{2/r}}\frac{a^{1+1/r}}{1+\frac{1}{r}}.$$

When

$$y = 0, \qquad x = \pm\frac{a^{1/r}}{2}\left[-\frac{a^{1-1/r}}{1-\frac{1}{r}} + \frac{1}{a^{2/r}}\frac{a^{1+1/r}}{1+\frac{1}{r}}\right]$$

$$= \pm\frac{1}{2}\left[\frac{-a}{1-\frac{1}{r}} + \frac{a}{1+\frac{1}{r}}\right] = \mp\frac{a}{r\left(1-\frac{1}{r^2}\right)} = \mp\frac{ar}{r^2-1}.$$

Evidently the + sign must be chosen. Hence, $PA = \dfrac{ar}{r^2-1}$, and the length of path QA is $\dfrac{ar^2}{r^2-1}$.

The north-going pack is now at B, where $PB = PA$. Taking PA as y-axis and PB as x-axis, equations (1) and (2) again hold good. But this time, when

$$x = 0, \qquad y = \frac{ar}{r^2 - 1} = a_1, \qquad \frac{dy}{dx} = -1.$$

Hence, from (1),

$$-1 = \frac{2ca_1^{1/r}}{1 - c^2a_1^{2/r}}.$$

Whence,

$$c = \frac{1 \pm \sqrt{2}}{a_1^{1/r}}$$

and, from (2),

$$c' = \frac{a_1^{1-1/r}}{1 - \frac{1}{r}} - \frac{3 \pm 2\sqrt{2}}{a_1^{2/r}} \cdot \frac{a_1^{1/r+1}}{1 + \frac{1}{r}}.$$

Hence, from (2),

$$2(1 \pm \sqrt{2})x = \frac{y^{1-1/r}}{1 - \frac{1}{r}}a_1^{1/r} - \frac{3 \pm 2\sqrt{2}}{a_1^{1/r}}\frac{y^{1+1/r}}{1 + \frac{1}{r}} - \frac{a_1}{1 - \frac{1}{r}} + \frac{(3 \pm 2\sqrt{2})a_1}{1 + \frac{1}{r}}.$$

When

$$y = 0, \qquad x = a_1\frac{(2 \pm 2\sqrt{2}) - (\pm 2\sqrt{2} + 4)\frac{1}{r}}{\left(1 - \frac{1}{r^2}\right)2(1 \pm \sqrt{2})} = \frac{a_1}{1 - \frac{1}{r^2}}\left\{1 \mp \frac{\sqrt{2}}{r}\right\}.$$

Hence,

$$PC = \frac{a_1}{1 - \frac{1}{r^2}}\left\{1 + \frac{\sqrt{2}}{r}\right\}.$$

Hence, the length of the path AC is

$$\frac{a_1r}{1 - \frac{1}{r^2}}\left\{1 + \frac{\sqrt{2}}{r}\right\}.$$

The length of each successive curve of pursuit will be found in the same way by multiplying the distance of the dog from P at starting, by

$$\frac{r}{1 - \frac{1}{r^2}}\left\{1 + \frac{\sqrt{2}}{r}\right\}.$$

Hence S, the total distance travelled by the dog, is

$$S = r\left[\frac{ar}{r^2 - 1}\left\{1 + \frac{r(r + \sqrt{2})}{r^2 - 1}\left(1 + \frac{r(r + \sqrt{2})}{r^2 - 1}\left(\left(\left(\cdots\right)\right)\right)\right)\right\}\right]$$

$$= \frac{ar^2}{r^2 - 1}\{1 + k + k^2 + \cdots k^{n-1}\}, \qquad \text{where} \qquad k = \frac{r(r + \sqrt{2})}{r^2 - 1}.$$

$$\text{Hence, } S = \frac{ar^2}{r^2 - 1}\,\frac{1 - k^n}{1 - k} = \frac{ar^2}{(r^2 - 1)^n}\,\frac{(r^2 - 1)^n - (r^2 + \sqrt{2}r)^n}{r\sqrt{2} + 1}.$$